



# basic education

Department:  
Basic Education  
**REPUBLIC OF SOUTH AFRICA**

## **NATIONAL SENIOR CERTIFICATE**

**GRADE 12**

**MATHEMATICS P1**

**NOVEMBER 2011**

**MARKS: 150**

**TIME: 3 hours**

**This question paper consists of 8 pages, 1 diagram sheet and 1 information sheet.**



**INSTRUCTIONS AND INFORMATION**

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. ONE diagram sheet for answering QUESTION 12.2 is attached at the end of this question paper. Write your centre number and examination number on this sheet in the spaces provided and insert the page inside the back cover of your ANSWER BOOK.
9. An information sheet, with formulae, is included at the end of the question paper.
10. Number the answers correctly according to the numbering system used in this question paper.
11. Write neatly and legibly.

**QUESTION 1**1.1 Solve for  $x$ :

1.1.1  $x(x+1) = 6$  (3)

1.1.2  $3x^2 - 4x = 8$  (4)

1.1.3  $4x^2 + 1 \geq 5x$  (4)

1.2 Consider the equation:  $x^2 + 5xy + 6y^2 = 0$ 

1.2.1 Calculate the values of the ratio  $\frac{x}{y}$ . (3)

1.2.2 Hence, calculate the values of  $x$  and  $y$  if  $x + y = 8$ . (5)  
**[19]**

**QUESTION 2**2.1 Given the sequence: 4 ;  $x$  ; 32Determine the value(s) of  $x$  if the sequence is:

2.1.1 Arithmetic (2)

2.1.2 Geometric (3)

2.2 Determine the value of  $P$  if  $P = \sum_{k=1}^{13} 3^{k-5}$  (4)

2.3 Prove that for any arithmetic sequence of which the first term is  $a$  and the constant difference is  $d$ , the sum to  $n$  terms can be expressed as  $S_n = \frac{n}{2}(2a + (n-1)d)$ . (4)  
**[13]**

**QUESTION 3**

The following sequence is a combination of an arithmetic and a geometric sequence:

$$3 ; 3 ; 9 ; 6 ; 15 ; 12 ; \dots$$

3.1 Write down the next TWO terms. (2)

3.2 Calculate  $T_{52} - T_{51}$ . (5)

3.3 Prove that ALL the terms of this infinite sequence will be divisible by 3. (2)  
**[9]**



**QUESTION 4**

A quadratic pattern has a second term equal to 1, a third term equal to  $-6$  and a fifth term equal to  $-14$ .

- 4.1 Calculate the second difference of this quadratic pattern. (5)
- 4.2 Hence, or otherwise, calculate the first term of the pattern. (2)
- [7]

**QUESTION 5**

- 5.1 Consider the function:  $f(x) = \frac{-6}{x-3} - 1$
- 5.1.1 Calculate the coordinates of the  $y$ -intercept of  $f$ . (2)
- 5.1.2 Calculate the coordinates of the  $x$ -intercept of  $f$ . (3)
- 5.1.3 Sketch the graph of  $f$  in your ANSWER BOOK, showing clearly the asymptotes and the intercepts with the axes. (4)
- 5.1.4 For which values of  $x$  is  $f(x) > 0$ ? (2)
- 5.1.5 Calculate the average gradient of  $f$  between  $x = -2$  and  $x = 0$ . (4)
- 5.2 Draw a sketch graph of  $y = ax^2 + bx + c$ , where  $a < 0$ ,  $b < 0$ ,  $c < 0$  and  $ax^2 + bx + c = 0$  has only ONE solution. (4)
- [19]

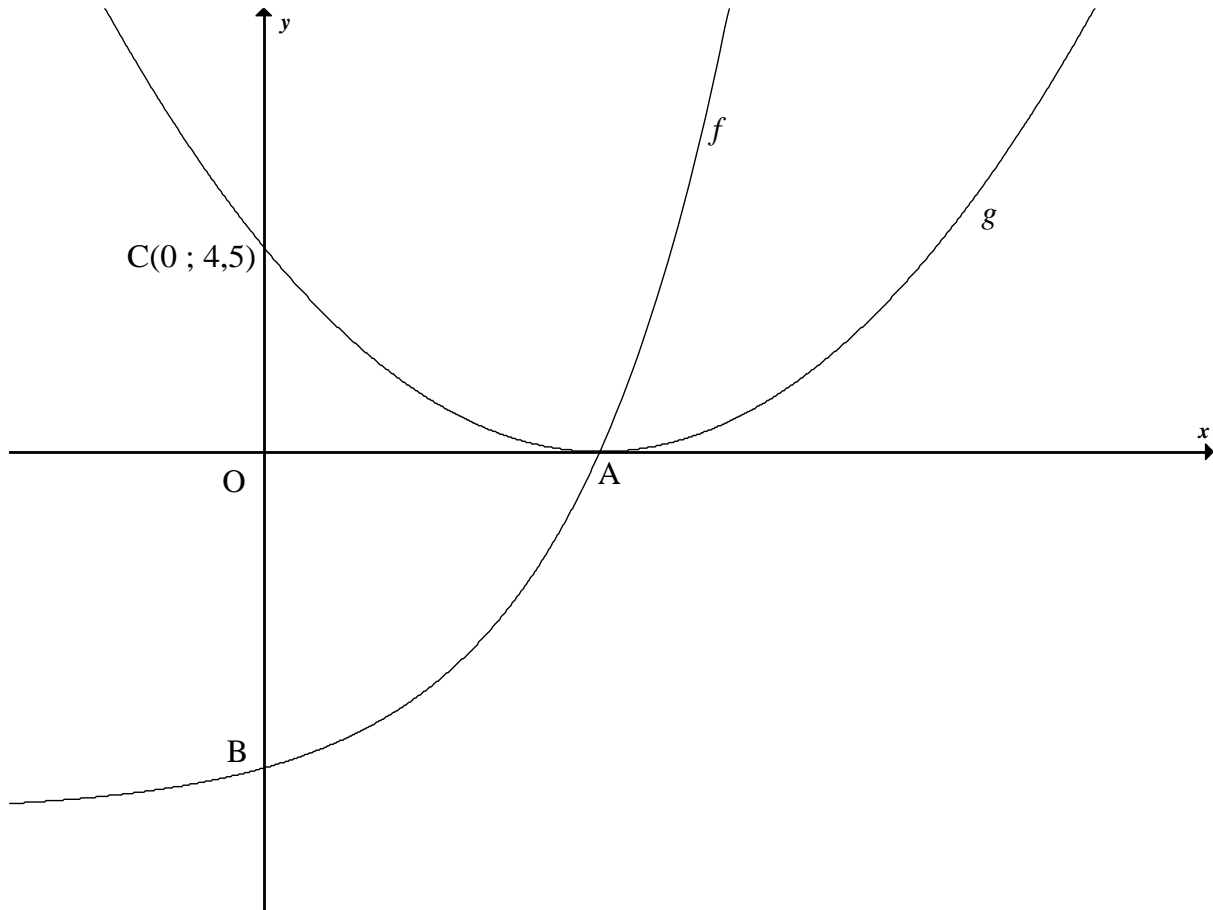


**QUESTION 6**

The graphs of  $f(x) = 2^x - 8$  and  $g(x) = ax^2 + bx + c$  are sketched below.

B and C(0 ; 4,5) are the y-intercepts of the graphs of  $f$  and  $g$  respectively.

The two graphs intersect at A, which is the turning point of the graph of  $g$  and the x-intercept of the graphs of  $f$  and  $g$ .



- 6.1 Determine the coordinates of A and B. (4)
  - 6.2 Write down an equation of the asymptote of the graph of  $f$ . (1)
  - 6.3 Determine an equation of  $h$  if  $h(x) = f(2x) + 8$ . (2)
  - 6.4 Determine an equation of  $h^{-1}$  in the form  $y = \dots$  (2)
  - 6.5 Write down an equation of  $p$ , if  $p$  is the reflection of  $h^{-1}$  about the  $x$ -axis. (1)
  - 6.6 Calculate  $\sum_{k=0}^3 g(k) - \sum_{k=4}^5 g(k)$ . Show ALL your working. (4)
- [14]**



**QUESTION 7**

- 7.1 How many years will it take for an article to depreciate to half its value according to the reducing-balance method at 7% per annum? (4)
- 7.2 Two friends each receive an amount of R6 000 to invest for a period of 5 years. They invest the money as follows:
- Radesh: 8,5% per annum simple interest. At the end of the 5 years, Radesh will receive a bonus of exactly 5% of the principal amount.
  - Thandi: 8% per annum compounded quarterly.
- Who will have the bigger investment after 5 years? Justify your answer with appropriate calculations. (6)
- 7.3 Nicky opened a savings account with a single deposit of R1 000 on 1 April 2011. She then makes 18 monthly deposits of R700 at the end of every month. Her first payment is made on 30 April 2011 and her last payment on 30 September 2012. The account earns interest at 15% per annum compounded monthly.
- Determine the amount that should be in her savings account immediately after her last deposit is made (that is on 30 September 2012). (6)
- [16]**

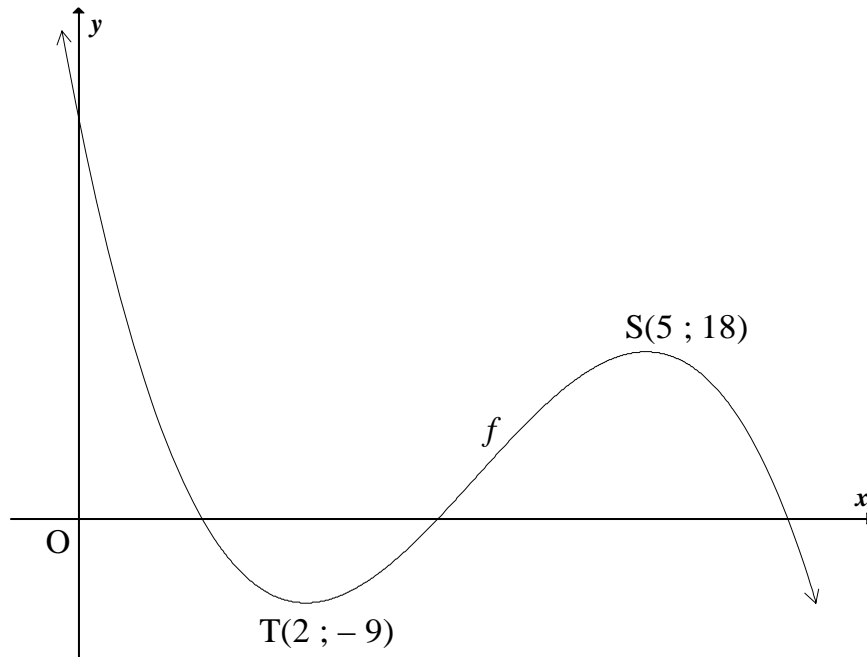
**QUESTION 8**

- 8.1 Determine  $f'(x)$  from first principles if  $f(x) = -4x^2$ . (5)
- 8.2 Evaluate:
- 8.2.1  $\frac{dy}{dx}$  if  $y = \frac{3}{2x} - \frac{x^2}{2}$  (3)
- 8.2.2  $f'(1)$  if  $f(x) = (7x+1)^2$  (4)
- [12]**



**QUESTION 9**

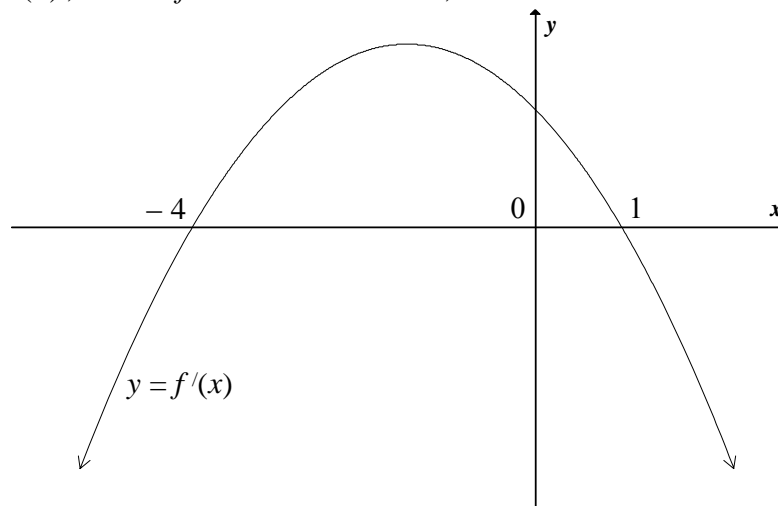
The function  $f(x) = -2x^3 + ax^2 + bx + c$  is sketched below.  
The turning points of the graph of  $f$  are  $T(2 ; -9)$  and  $S(5 ; 18)$ .



- 9.1 Show that  $a = 21$  ,  $b = -60$  and  $c = 43$ . (7)
  - 9.2 Determine an equation of the tangent to the graph of  $f$  at  $x = 1$ . (5)
  - 9.3 Determine the  $x$ -value at which the graph of  $f$  has a point of inflection. (2)
- [14]**

**QUESTION 10**

The graph of  $y = f'(x)$ , where  $f$  is a cubic function, is sketched below.



Use the graph to answer the following questions:

- 10.1 For which values of  $x$  is the graph of  $y = f'(x)$  decreasing? (1)
  - 10.2 At which value of  $x$  does the graph of  $f$  have a local minimum? Give reasons for your answer. (3)
- [4]**



**QUESTION 11**

Water is flowing into a tank at a rate of 5 litres per minute. At the same time water flows out of the tank at a rate of  $k$  litres per minute. The volume (in litres) of water in the tank at time  $t$  (in minutes) is given by the formula  $V(t) = 100 - 4t$ .

- 11.1 What is the initial volume of the water in the tank? (1)
- 11.2 Write down TWO different expressions for the rate of change of the volume of water in the tank. (3)
- 11.3 Determine the value of  $k$  (that is, the rate at which water flows out of the tank). (2)
- [6]**

**QUESTION 12**

A school is planning a trip for 500 learners. The company that will be providing the transport has two types of buses, red buses and blue buses, available.

- Each red bus has 50 seats and each blue bus has 25 seats.
- The company has at most 15 bus drivers available.
- There are at most 8 blue buses available.

Let the number of red buses hired by the school be  $x$  and the number of blue buses hired by the school be  $y$ .

- 12.1 Write down ALL the constraints, in terms of  $x$  and  $y$ , to represent the above information. (6)
- 12.2 Represent the constraints graphically on the attached DIAGRAM SHEET. Clearly indicate the feasible region. (4)
- 12.3 The cost of hiring a red bus is R600 for the day and the cost of hiring a blue bus is R300 for the day. Write down the total transport cost. (1)
- 12.4 12.4.1 Determine ALL possible values of  $x$  and  $y$  so that the cost will be a minimum. (3)
- 12.4.2 Calculate the minimum cost of hiring the buses. (2)
- 12.5 If exactly 12 bus drivers are to be used, determine the number of each type of bus which the school will now need to still ensure minimum cost. (1)
- [17]**

**TOTAL: 150**





**CENTRE NUMBER:**

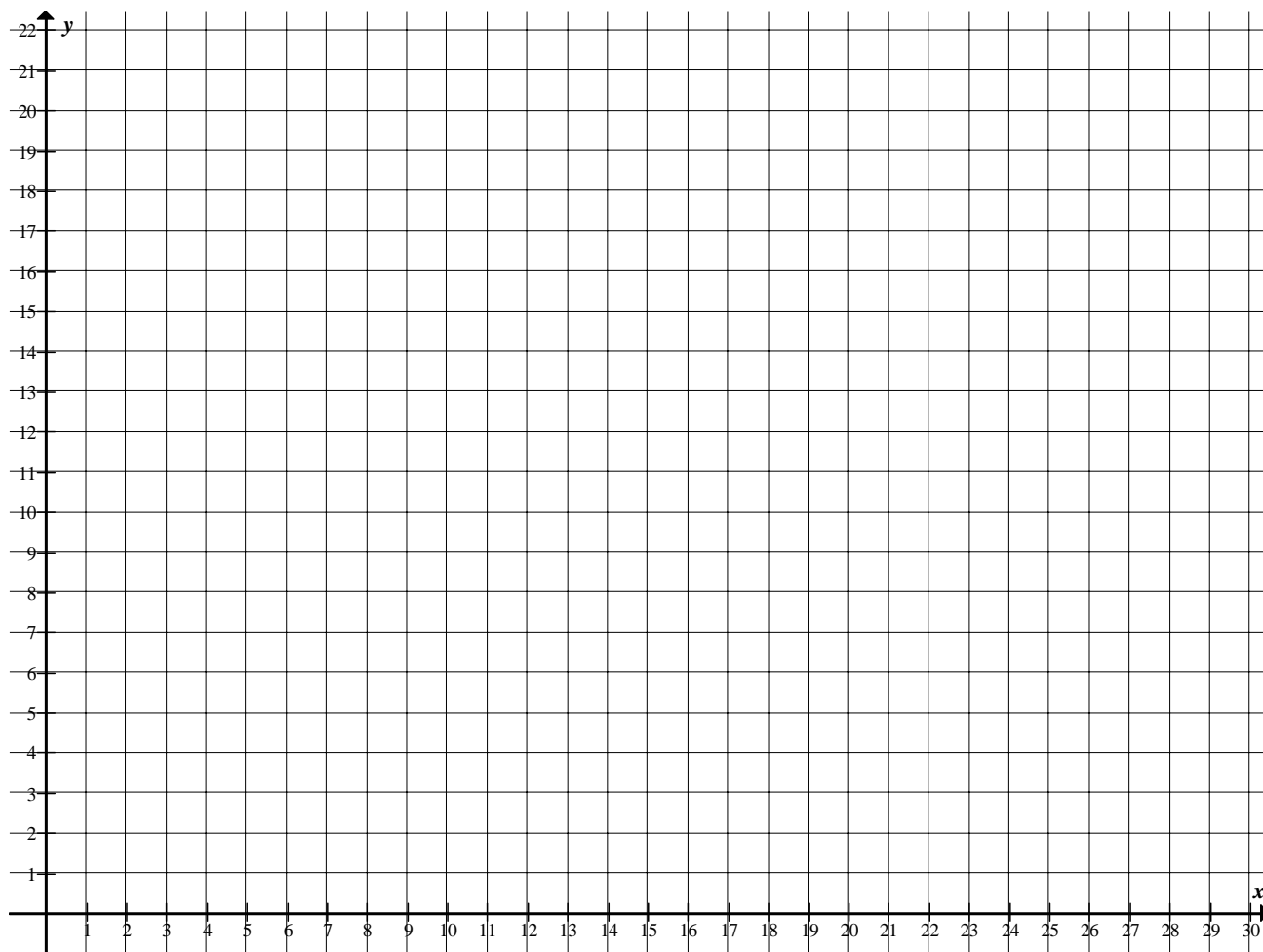
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**DIAGRAM SHEET 1**

**QUESTION 12.2**



**INFORMATION SHEET: MATHEMATICS**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta + y \sin \theta; y \cos \theta - x \sin \theta)$$

$$(x; y) \rightarrow (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

