

# education

Department of Education  
REPUBLIC OF SOUTH AFRICA

## SENIOR CERTIFICATE EXAMINATION - 2006

**MATHEMATICS PAPER 2  
GEOMETRY**

**HIGHER GRADE**

**OCTOBER/NOVEMBER 2006**

**301-1/2E**

**MATHEMATICS HG: Paper 2**

**MARKS: 200**



**301 1 2E**

**HG**

**TIME: 3 hours**

**This question paper consists of 11 pages, a formula sheet and 5 diagram sheets.**



**INSTRUCTIONS AND INFORMATION**

Read the following instructions carefully before answering the questions:

1. This question paper consists of 9 questions, a formula sheet and 5 diagram sheets.
2. Use the formula sheet to answer this question paper.
3. Detach the diagram sheets from the question paper and place them inside your ANSWER BOOK.
4. The diagrams are not drawn to scale.
5. Answer ALL the questions.
6. Number ALL the answers correctly and clearly.
7. ALL the necessary calculations must be shown.
8. Non-programmable calculators may be used, unless otherwise stated.
9. The number of decimal digits to which answers must be rounded off will be stated in the question where necessary.

## ANALYTICAL GEOMETRY

**NOTE:** – USE ANALYTICAL METHODS IN THIS SECTION.  
 – CONSTRUCTION AND MEASUREMENT METHODS MAY NOT BE USED.

## QUESTION 1

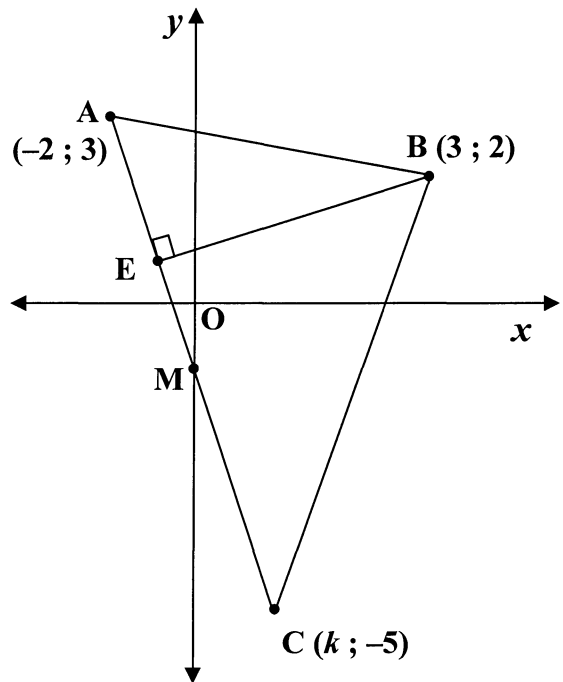
In the diagram alongside,

$A(-2 ; 3)$ ,  $B(3 ; 2)$  and  $C(k ; -5)$

are three points in a Cartesian plane.

$M$ , the midpoint of  $AC$ , lies on the  $y$ -axis.

$BE \perp AC$ , with  $E$  a point on  $AC$ .



- 1.1 Write down the value of  $k$ . (1)
- 1.2 Calculate the size of  $\hat{A}$ , rounded off to ONE decimal digit. (7)
- 1.3 Determine the equation of altitude  $BE$ . (4)
- 1.4 Determine the co-ordinates of  $E$ . (6)
- 1.5 Calculate the area of  $\triangle ABM$ . (6)

[24]

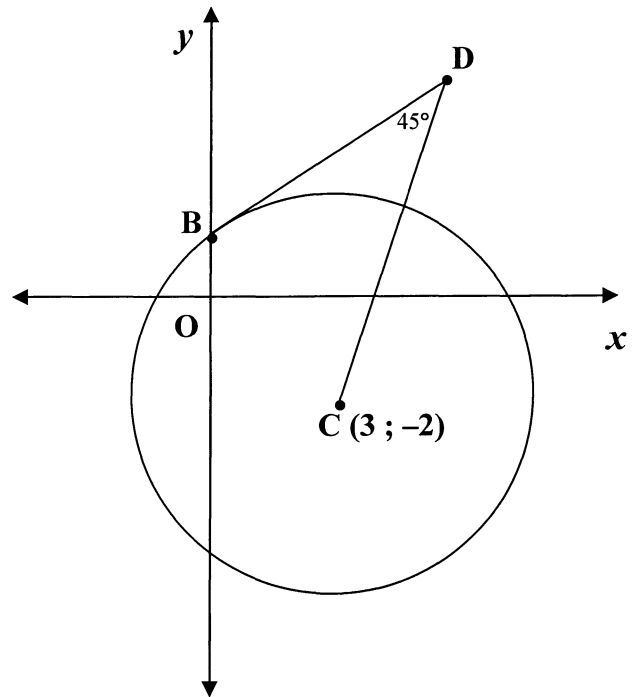
**QUESTION 2**

2.1 In the diagram alongside, BD is a tangent to the circle at point B, which lies on the y-axis.

The centre of the circle is C (3 ; -2).

The equation of tangent BD is given by  $3x - 4y + 8 = 0$

$$\hat{BDC} = 45^\circ$$



2.1.1 Determine the co-ordinates of B. (2)

2.1.2 Show that  $x^2 - 6x + y^2 + 4y - 12 = 0$  is the equation of the circle. (4)

2.1.3 Determine the value(s) of q if  $x + q = 0$  is the equation of a tangent to the circle. (4)

2.1.4 (a) Write down the length of BD. (1)

(b) Hence, determine the co-ordinates of D. (6)

2.1.5 Determine whether points E (2 ; -9), C and D are collinear. (3)

2.2 Determine the equation of the locus of point P (x ; y), if the distance from P to R (1 ; -4) is equal to two times the distance from P to T (-2 ; -1). (7)

[27]



**TRIGONOMETRY****QUESTION 3**

Answer this question without the use of a calculator.

3.1 If  $\sec 751^\circ = k$ , express each of the following in terms of  $k$ :

3.1.1  $\cos 31^\circ$  (2)

3.1.2  $2 \operatorname{cosec} (-121^\circ)$  (3)

3.1.3  $\tan 329^\circ$  (3)

3.2 Simplify:

$$\sqrt{\tan(-207^\circ) \cdot \cot 333^\circ - \frac{\sin^2(x - 360^\circ) \cdot \operatorname{cosec}(x - 90^\circ)}{\cos x}}$$
 (10)

[18]

**QUESTION 4**

Given:  $f(x) = \cos \frac{1}{2}x$  and  $g(x) = \sin(x + 60^\circ)$

4.1 Solve for  $x$  if  $\sin(x + 60^\circ) = \cos \frac{1}{2}x$  and  $x \in [-60^\circ ; 300^\circ]$  (7)

4.2 Use the set of axes provided on the diagram sheet to draw sketch graphs of the curves of  $f$  and  $g$  for  $x \in [-60^\circ ; 300^\circ]$ . Show clearly the co-ordinates of all turning points and end points and the intercepts with the axes. (10)

4.3 Use the solution obtained in QUESTION 4.1 as well as the graph drawn in QUESTION 4.2 to determine the value(s) of  $x \in [-60^\circ ; 300^\circ]$  for which:

4.3.1  $f(x) < g(x)$  (3)

4.3.2  $f(x) \cdot g(x) \leq 0$  (3)

[23]

**QUESTION 5**

5.1 Determine the general solution of the equation:

$$2 \sin x + \operatorname{cosec} x - 3 = 0 \quad (9)$$

5.2 5.2.1 Write down an expression for  $\cos 2\theta$  in terms of  $\cos \theta$ . (1)

5.2.2 Prove the identity:

$$2 \cos \theta \cdot \cos 2\theta + \sec \theta \cdot \sin^2 2\theta = 2 \cos \theta \quad (7)$$

5.3 5.3.1 Write down an expression for  $\tan (A + B)$  in terms of  $\tan A$  and  $\tan B$  (1)

5.3.2 Hence, if  $\tan (A + B) = 4$  and  $\tan A = 1$ , calculate, without the use of a calculator, the numerical value of  $\tan B$ . (4)

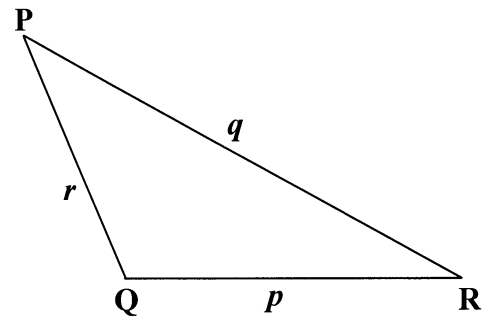
**[22]**

**QUESTION 6**

6.1 In the diagram alongside  $\Delta PQR$  is shown.

Use the diagram on the diagram sheet or redraw the diagram in your answer book, to prove that:

$$\frac{\sin Q}{q} = \frac{\sin P}{p}$$



(3)

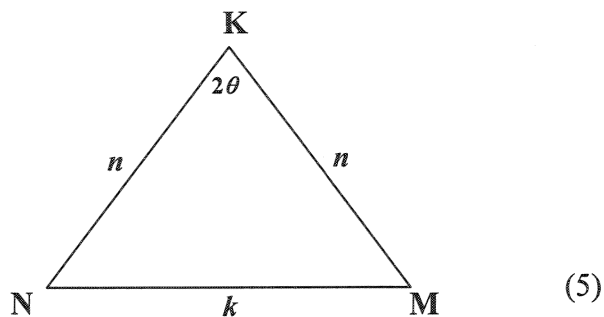


6.2 In the diagram alongside,  $\Delta KMN$  is given with  $\hat{K} = 2\theta$ , and

$KM = KN = n$  units

$NM = k$  units

Prove that  $k = 2n \cdot \sin \theta$



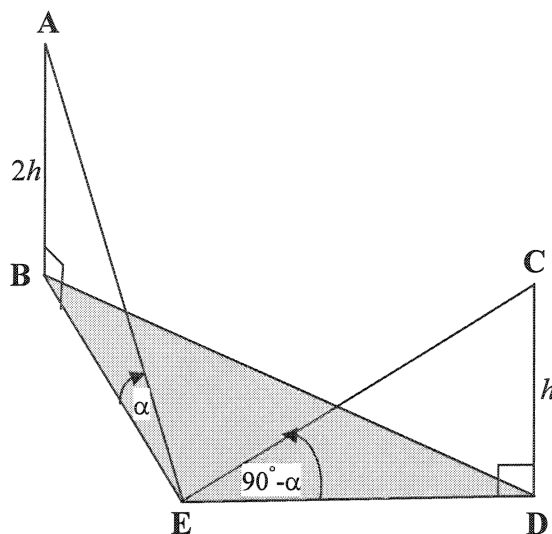
6.3 In the diagram alongside, points B, D and E lie in the same horizontal plane with  $\hat{BED} = 120^\circ$ .

AB and CD are two vertical towers.

$AB = 2CD = 2h$  metres.

The angle of elevation of A from point E is  $\alpha$ .

The angle of elevation of C from point E is  $(90^\circ - \alpha)$ .



6.3.1 Determine the length of BE in terms of  $h$  and  $\alpha$ . (2)

6.3.2 Prove that the distance between the bases of the two towers is given by:

$$BD = \frac{h \sqrt{\tan^4 \alpha + 2 \tan^2 \alpha + 4}}{\tan \alpha} \tag{8}$$

6.3.3 Hence, determine the height of tower CD, rounded off to the nearest metre, if  $\alpha = 48^\circ$  and  $BD = 509$  m. (4)  
**[22]**



**EUCLIDEAN GEOMETRY**

**NOTE:**

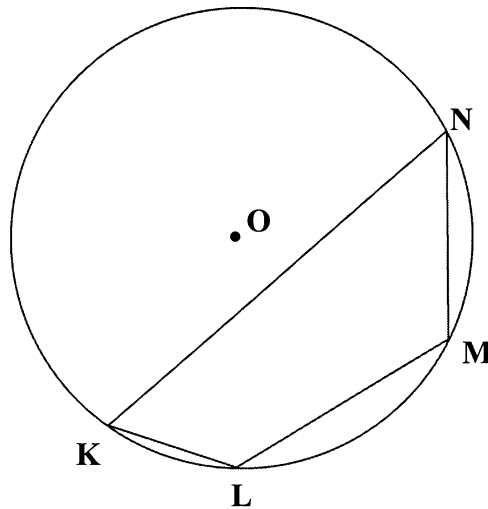
- **DIAGRAMS FOR PROVING THEORY MAY BE USED ON THE DIAGRAM SHEETS OR REDRAWN IN YOUR ANSWER BOOK.**
- **DETACH THE DIAGRAM SHEETS FROM THE QUESTION PAPER AND PLACE THEM IN YOUR ANSWER BOOK.**
- **GIVE A REASON FOR EACH STATEMENT, UNLESS OTHERWISE STATED.**

**QUESTION 7**

7.1 In the diagram below, circle KLMN is drawn.

Use the diagram on the diagram sheet or redraw the diagram in your answer book to prove the theorem which states that:

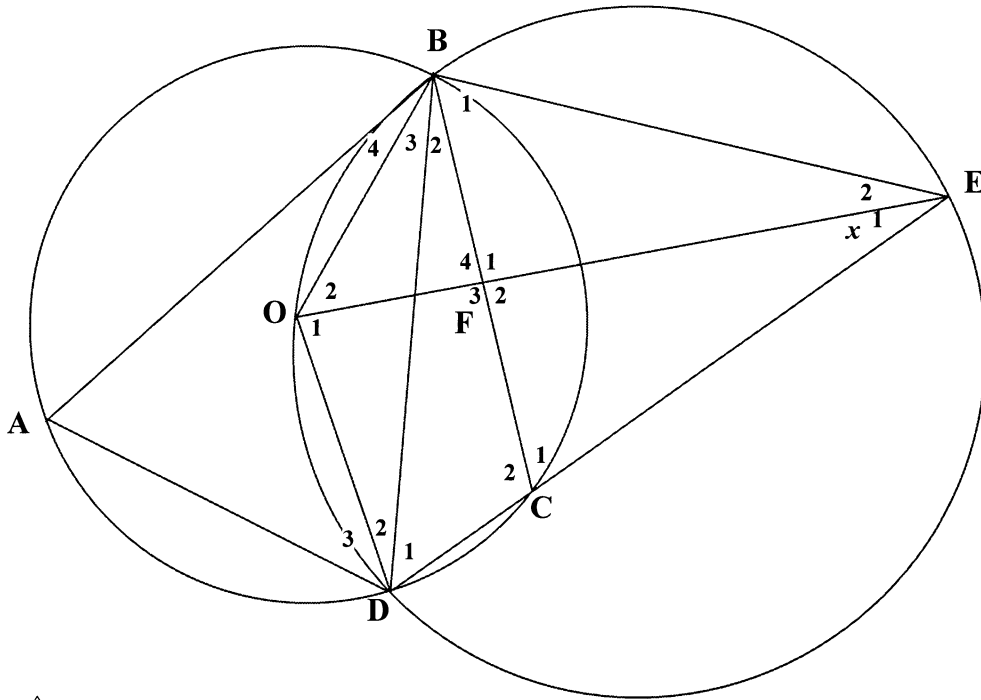
If **O** is the centre of the circle, then  $\hat{L} + \hat{N} = 180^\circ$



(6)



7.2 In the diagram below, O is the centre of circle ABCD.  
 DC is produced to meet circle BODE at E.  
 OE intersects BC at F.



Let  $\hat{E}_1 = x$

7.2.1 Determine the size of  $\hat{A}$  in terms of  $x$ . (6)

7.2.2 Prove that:

(a)  $BE = EC$  (7)

(b) BE is NOT a tangent to circle ABCD (3)

[22]

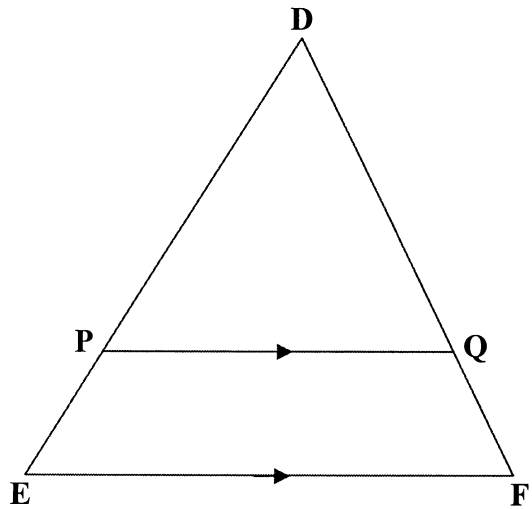


**QUESTION 8**

- 8.1 In the diagram alongside,  
P is a point on DE and Q is a point on DF  
of  $\triangle DEF$ .

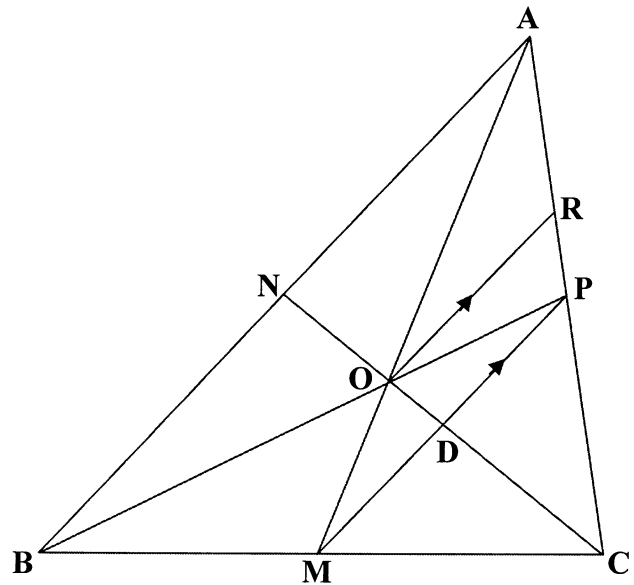
Use the diagram on the diagram sheet or  
redraw the diagram in your answer book to  
prove the theorem which states that:

If  $PQ \parallel EF$ , then  $\frac{DP}{DE} = \frac{DQ}{DF}$



(7)

- 8.2 In the diagram alongside,  
medians AM and CN of  $\triangle ABC$ , intersect at O.  
BO is produced to cut AC at P.  
MP and CN intersect at D.  
OR  $\parallel$  MP with R on AC.



- 8.2.1 Calculate, with reasons, the numerical value of  $\frac{ND}{NC}$  (6)

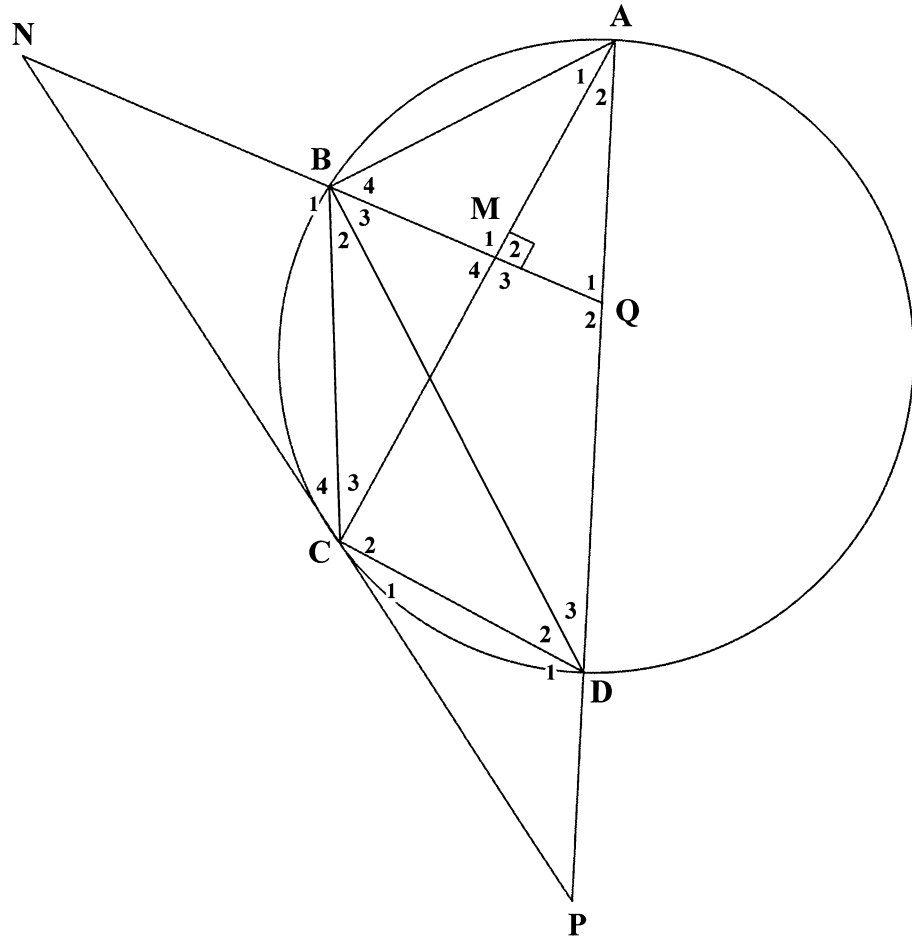
- 8.2.2 Use  $AO : AM = 2 : 3$ , to calculate the numerical value of  $\frac{RP}{PC}$  (5)

[18]



**QUESTION 9**

In the diagram below AD is the diameter of circle ABCD.  
 AD is produced to meet tangent NCP at P.  
 Straight line NB is produced to Q and intersects  
 AC at M with Q on ADP.  
 $AC \perp NQ$  at M.



- 9.1 Prove that  $NQ \parallel CD$ . (3)
- 9.2 Prove that  $ANCQ$  is a cyclic quadrilateral. (4)
- 9.3 9.3.1 Prove that  $\triangle PCD \sim \triangle PAC$  (3)
- 9.3.2 Hence, complete the statement:  $PC^2 = \dots$  (2)
- 9.4 Prove that  $BC^2 = CD \cdot NB$  (7)
- 9.5 If it is further given that  $PC = MC$ , prove that

$$1 - \frac{BM^2}{BC^2} = \frac{AP \cdot DP}{CD \cdot NB} \quad (5) \quad [24]$$

**TOTAL: 200**





**Mathematics Formula Sheet (HG and SG)**  
**Wiskundeformuleblad (HG en SG)**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2} (a + T_n)$$

$$S_n = \frac{n}{2} (a + \ell)$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$T_n = a \cdot r^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad (r \neq 1)$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1 - r} \quad (|r| < 1)$$

$$A = P \left( 1 + \frac{r}{100} \right)^n$$

$$A = P \left( 1 - \frac{r}{100} \right)^n$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x_3 ; y_3) = \left( \frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2} \right)$$

$$x^2 + y^2 = r^2$$

$$(x - p)^2 + (y - q)^2 = r^2$$

$$\text{In } \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$







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**DEPARTEMENT VAN ONDERWYS**

**REPUBLIC OF SOUTH AFRICA**  
**REPUBLIEK VAN SUID-AFRIKA**

**SENIOR CERTIFICATE EXAMINATION/SENIORSERTIFIKAAT-EKSAMEN**  
**MATHEMATICS HG/WISKUNDE HG**  
**PAPER II/VRAESTEL II**  
**OCTOBER/OKTOBER/NOVEMBER 2006**

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**DIAGRAM SHEET/DIAGRAMVEL**

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**INSTRUCTION**

This diagram sheet must be handed in with your answer book. Ensure that your details are complete.

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**INSTRUKSIE**

Hierdie diagramvel moet saam met jou antwoordeboek ingelewer word. Maak seker dat jou besonderhede volledig ingevul is.

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**EXAMINATION NUMBER**  
**EKSAMENNOMMER**

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**CENTRE NUMBER**  
**SENTRUMNOMMER**

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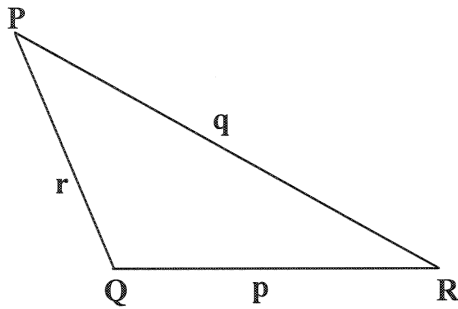




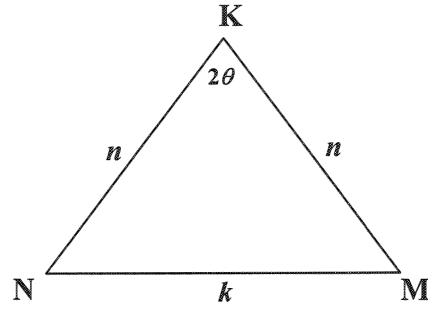
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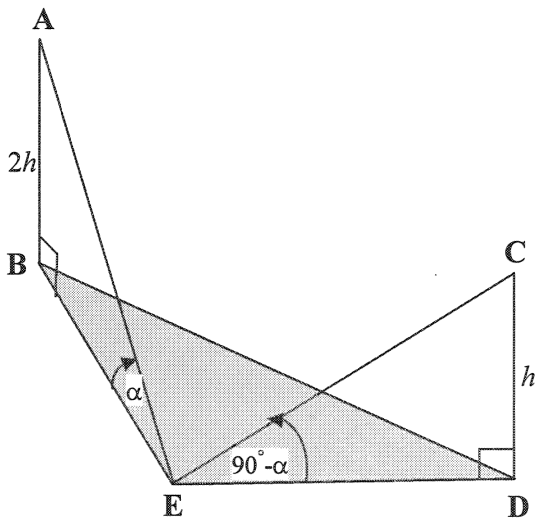
QUESTION 6.1 / VRAAG 6.1



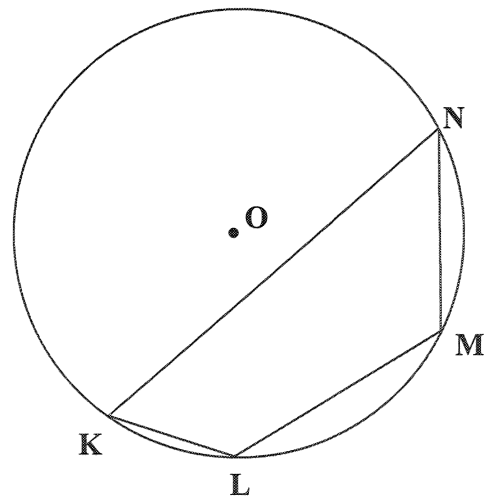
QUESTION 6.2 / VRAAG 6.2



QUESTION 6.3 / VRAAG 6.3



QUESTION 7.1 / VRAAG 7.1

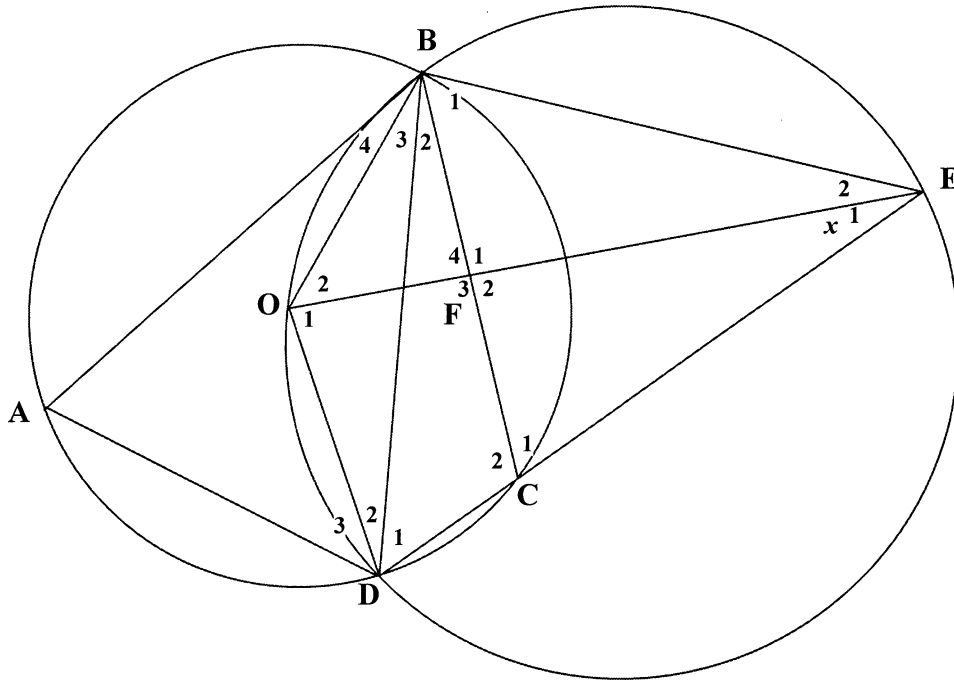




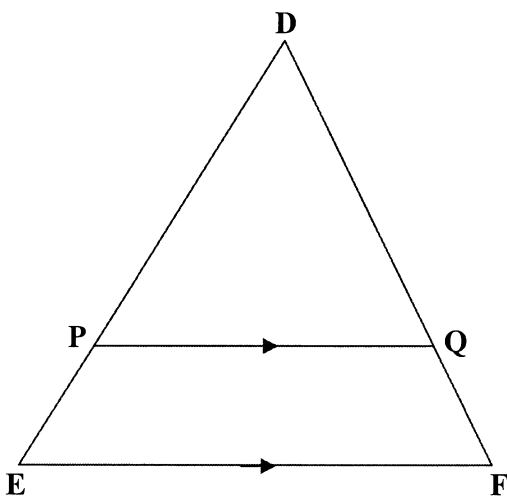
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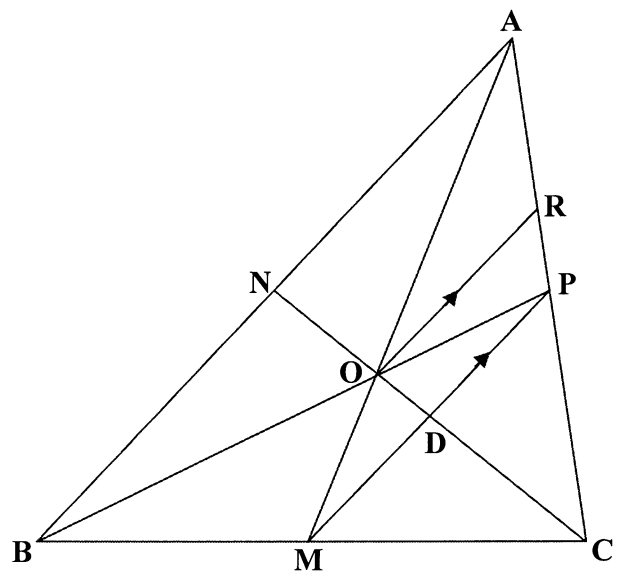
**QUESTION 7.2 / VRAAG 7.2**



**QUESTION 8.1 / VRAAG 8.1**



**QUESTION 8.2 / VRAAG 8.2**





EXAMINATION NUMBER  
EKSAMENNOMMER

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QUESTION 9 /VRAAG 9

